$$\boxed{\text{スベルドラップバランス}}$$

β平面で仮定

$$f = f_0 + \beta y \quad (\text{緯度}\theta_0\text{で}f_0)$$

線形化した運動方程式は

$$\frac{\partial u}{\partial t} - (f_0+\beta y)v = -g\frac{\partial \eta}{\partial x} \cdots ①$$

$$\frac{\partial v}{\partial t} + (f_0+\beta y)u = -g\frac{\partial \eta}{\partial y} \cdots ②$$

渦度方程式をつくる。 $\frac{\partial ②}{\partial x} - \frac{\partial ①}{\partial y}$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + (f_0+\beta y)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta v = 0$$

定常問題 $\frac{\partial}{\partial t} = 0$ を考えると

$$\beta v = -(f_0+\beta y)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$u = u_G + u'$, $v = v_G + v'$ のように
地衡流成分と非地衡流成分に分ける

$$\beta(v_G + v') = -(f_0+\beta y)\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)$$

ここで地衡流成分の水平発散

$\frac{\partial u_G}{\partial x} + \frac{\partial v_G}{\partial y} = 0$ を用いている

β, u', v' が小さいと仮定すると
(西岸境界流域以外を考えると)

$$\beta v_G = -f_0\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)$$

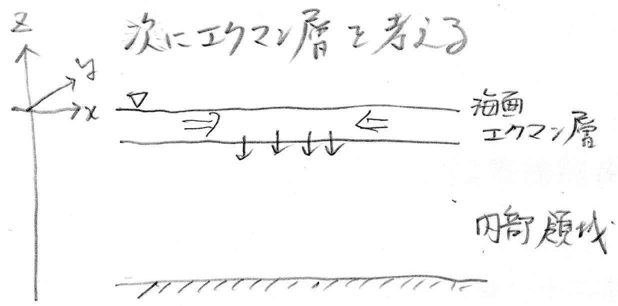
内部領域で積分　　$\bar{v}_G$ は鉛直平均した流速

$$\beta \bar{v}_G H = f_0\int \frac{\partial w'}{\partial z}dz$$

$$= f_0\left[w'(\text{エクマン層の下面}) - w'(\text{海底})\right]$$

$$= f_0 w'(\text{エクマン層の下面})$$

次にエクマン層を考える



緯度θ₀での運動方程式(定常なとき)
を考える

$$-f_0 v = -g\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}$$

$$f_0 u = -g\frac{\partial P}{\partial y} + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial z^2}$$

速度を地衡流成分と非地衡流成分に分ける

$u = u_G + u'$, $v = v_G + v'$

また u', v' は圧力(もしくは η)に依存
しない流れ エクマンらせんの流れで
あるとする ($P = P_G$ で $P' = 0$)

$$-f_0(v_G + v') = -g\frac{\partial P_G}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2}{\partial z^2}(u_G + u')$$

$$f_0(u_G + u') = -g\frac{\partial P_G}{\partial y} + \frac{\mu}{\rho}\frac{\partial^2}{\partial z^2}(v_G + v')$$

地衡流成分の方程式

$$-f_0 v_G = -g\frac{\partial P_G}{\partial x} + \frac{\partial^2 u_G}{\partial z^2}$$

$$f_0 u_G = -g\frac{\partial P_G}{\partial y} + \frac{\partial^2 v_G}{\partial z^2}$$

↓ 鉛直平均流を考える ($\frac{\partial}{\partial z} = 0$)

$$-f_0 \bar{v}_G = -g\frac{\partial P_G}{\partial x}$$

$$f_0 \bar{u}_G = -g\frac{\partial P_G}{\partial y}$$

エクマン層の方程式

$$-f_0 v' = \frac{\mu}{\rho}\frac{\partial^2 u'}{\partial z^2}$$

$$f_0 u' = \frac{\mu}{\rho}\frac{\partial^2 v'}{\partial z^2}$$

エクマン層にわたって積分
(全層にわたって積分しても同じ)

$$-f_0 V' = \frac{\mu}{\rho}\frac{\partial u'}{\partial z}\bigg|_{z=0}$$

$$f_0 U' = \frac{\mu}{\rho}\frac{\partial v'}{\partial z}\bigg|_{z=0}$$

$z=0$ で $\tau_x = \mu\frac{\partial u'}{\partial z}$, $\tau_y = \mu\frac{\partial v'}{\partial z}$ なので

$$V' = -\frac{1}{f_0\rho}\tau_x, \quad U' = \frac{1}{f_0\rho}\tau_y$$

$$\frac{\partial U'}{\partial x} + \frac{\partial V'}{\partial y} = +\frac{1}{f_0\rho}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

$$\frac{\partial U'}{\partial x} + \frac{\partial V'}{\partial y} = -\int_{\text{エクマン層の下面}}^{\text{海面}} \frac{\partial w'}{\partial z}dz$$

$$= -\left[w'(\text{海面}) - w'(\text{エクマン層の下面})\right]$$

$$= +w'(\text{エクマン層の下面})$$

$$\beta \bar{v}_G H = f_0 \cdot \frac{1}{f_0\rho}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right) \Rightarrow \bar{v}_G = \frac{1}{\beta\rho H}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$