ABSTRACT

The most part of energy losses in a boiler is due to the heat released by the exhaust flue gas to atmosphere. The released heat consists of sensible and latent one. Recently, for a biological and environmental safety, a clean fuel such as a natural gas is widely used in the boiler. As the clean fuel includes a lot of hydrogen instead of carbon, the exhaust flue gas includes a lot of steam accompanying with the latent heat. So the latent heat recovery from the flue gas is very important to improve the boiler efficiency.

A prediction code for the heat exchanger to recover the latent heat in the flue gas has been proposed. In the present study, the prediction code was used on the parametric study of the heat exchanger design for the latent heat recovery. The thermal-hydraulic behavior was calculated for several kinds of heat exchangers using finned tubes or bare tubes. The calculation result indicated that the most compact heat exchanger was that using the bare tube of small diameter. So the compact countercurrent cross-flow heat exchanger using bare tubes of SUS304 was designed and constructed to prove its high ability. The outer and inner diameter of the bare tube were 10.5 and 8.5mm, respectively. The bare tubes were arranged in a staggered bank of 10-9 rows and 40 stages.

The experimental study varying the air ratio of flue gas, feed water temperature and flow rate was conducted. The temperature distributions of water and flue gas in the heat exchanger were measured. The pressure loss and the total amount of condensate generated in the heat exchanger were also measured. The experimental results for the temperature distributions of water and flue gas in the heat exchanger with bare tubes of small diameter agreed well with the prediction. The proposed compact heat exchanger using small tubes was considered to be preferable for the latent heat recovery from the flue gas and the prediction code was useful for the design of the compact heat exchanger.

NOMENCLATURE

C: mass concentration per fluid of a unit volume
C_p: specific heat
d: outer diameter of tube
d_i: inner diameter of tube
D: mass diffusivity
g: acceleration due to gravity
h_v: heat transfer coefficient
h_c: mass transfer coefficient
L_w: latent heat
Nu: Nusselt number \[ = \frac{h_v d}{\lambda} \]
Nr: total number of stages
P: pressure
q: heat flux
R: relative humidity of combustion air
Pr: Prandtl number \[ = \frac{\nu \kappa}{\lambda} \]
Re: Reynolds number \[ = \frac{u d}{\nu} \]
S_1: spanwise pitch
S_2: flow-directional pitch
Sh: Sherwood number \[ = \frac{h_c d}{D} \]
Sc: Schmidt number \[ = \frac{\nu}{D} \]
T: temperature
u: velocity at minimum flow area
V: volumetric flow rate
w: mass concentration per fluid of an unit mass
x: relative humidity of air
\( \kappa \): thermal diffusivity \[ = \frac{\lambda}{(\rho C_p)} \]
\( \lambda \): heat conductivity
\( \mu \): air ratio
v: kinematic viscosity
\( \rho \): density

subscript
INTRODUCTION

The most part of energy losses in a boiler is due to the heat released by the exhaust flue gas to atmosphere. The released heat consists of sensible and latent one. Recently, for a biological and environmental safety, a clean fuel such as a natural gas is widely used in the boiler. As the clean fuel includes a lot of hydrogen instead of carbon, the exhaust flue gas includes a lot of steam accompanied with the latent heat. So the latent heat recovery from the flue gas is very important to improve the boiler efficiency.

In the previous studies [Osakabe, 1998a, 1999a, 1999b], condensation heat transfer on horizontal stainless steel tubes has been investigated experimentally by using the actual flue gas from a natural gas boiler. The experiments were conducted using single and 2 stages of tubes at different air ratios and steam mass concentrations of the flue gas in a wide range of tube wall temperature. The condensation heat transfer was well predicted with a simple analogy correlation in the high wall temperature region and the low steam mass concentration typical in air combustion. In the low wall temperature region less than 30°C or the high steam mass concentration typical in oxygen combustion, the total heat transfer was higher than that predicted by the simple analogy correlation. The condensation heat transfer on spirally finned tubes of single stage was also investigated experimentally and theoretically [Osakabe, 1998b].

The analogous heat and mass transfer model for the finned tube was proposed.

Also in the previous studies [Osakabe, 1999c, 2000], a prediction code for the heat exchanger to recover the latent heat in the flue gas was proposed. In the prediction, the flue gas was treated as a mixture of CO₂, CO, O₂, N₂ and H₂O, and the heat and mass balance calculation along the flow direction of flue gas was adapted. The heat and mass transfer on tubes was evaluated with a simple analogy correlation. For the finned tubes, the conclusion heat transfer on spirally finned tubes of single stage was also investigated experimentally and theoretically [Osakabe, 1999b]. The fin efficiency at the condensing region was significantly lower than that at the dry region. The analogous heat and mass transfer model for the finned tube was proposed.

In the present study, the prediction code was used on the heat exchanger for the latent heat recovery. The thermal-hydraulic behavior was calculated for several kinds of heat exchangers using finned tubes or bare tubes. The calculation result indicated that the most compact heat exchanger was that using the bare tube of small diameter. So the compact countercurrent cross-flow heat exchanger using bare tubes of SUS304 was designed and constructed to prove its high ability. The outer and inner diameter of the bare tube were 10.5 and 8.5mm, respectively. The bare tubes were arranged in a staggered bank of 10-9 rows and 40 stages.

The experimental study varying the air ratio of flue gas, feed water temperature and flow rate was conducted. The temperature distributions of water and flue gas in the heat exchanger were measured with sheathed K-type thermocouples of 0.5 mm in diameter. The pressure loss and the total amount of condensate generated in the heat exchanger were also measured.

<table>
<thead>
<tr>
<th>Table 1. Composition of natural gas fuel (13A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH₄</td>
</tr>
<tr>
<td>C₂H₆</td>
</tr>
<tr>
<td>C₃H₈</td>
</tr>
<tr>
<td>C₄H₁₀</td>
</tr>
</tbody>
</table>

CONSTITUTIVE EQUATIONS FOR PREDICTION

Fuel combustion

The composition of natural gas fuel used in the test boiler is shown in Table 1. Volumetric concentrations of N₂, CO₂, O₂ and CO in the dry gas were measured by a gas analyzer. By using the measured concentration, the air ratio μ is calculated as

\[
\mu = \frac{N_2}{N_2 - \frac{0.79}{0.21} (O_2 - 0.5 \cdot CO)}
\]

(1)
The molar fraction of the carbon in the fuel of 1 mol can be calculated as,

\[
CCR = 1 \times 0.88 + 2 \times 0.058 + 3 \times 0.045 + 4 \times 0.017 \text{ mol}
\]

By using the volumetric flow rate of the fuel Vᵢ, the volumetric flow rate of the carbon dioxide and monoxide, V_COX, is

\[
V_{COX} = V_F \cdot CCR
\]

(2)
The volumetric flow rate of dry gas V_d is,

\[
V_d = \frac{V_{COX}}{CO_2 + CO}
\]

(3)
The molar fraction of the hydrogen corresponding to COₓ of 1 mol can be calculated as,

\[
CHR = (4 \times 0.88 + 6 \times 0.058 + 8 \times 0.045 + 10 \times 0.017) / CCR \text{ mol}
\]

Considering that the air flow rate necessary for the combustion of the fuel is V_dN₂/0.79, the volumetric fraction of H₂O in the flue gas can be estimated as

\[
H_2O = \frac{(CO_2 + CO) \cdot CHR/2 + N_2 \cdot R \cdot P_{sat}/(Pa \cdot 0.79)}{1 + (CO_2 + CO) \cdot CHR/2 + N_2 \cdot R \cdot P_{sat}/(Pa \cdot 0.79)}
\]

(4)
where P_{sat} is the saturation pressure of steam in combustion air.

The volumetric flow rate of wet gas, V_{wt}, is

\[
V_{wt} = \frac{V_d}{1 - H_2O}
\]

(5)
When the flue gas temperature is T_f°C, the steam mass

sub: subcooling, wt: wet gas
concentration $C_f$ per unit volume of flue gas is,

$$C_f = \frac{H_2O \cdot 18 + 273.15}{22.4 + 273.15 + T_f}$$  (6)

The steam mass concentration $w_f$ per unit mass of flue gas is

$$w_f = \frac{H_2O \cdot 18}{H_2O \cdot 18 + (1 - H_2O)(CO_2 \cdot 44 + CO \cdot 28 + N_2 \cdot 28 + O_2 \cdot 32)}$$  (7)

**Heat and mass transfer in gas side**

The total heat flux $q_T$ consists of the convection heat flux $q_V$ and the condensation heat flux $q_C$ as

$$q_T = q_V + q_C$$  (8)

The convection heat flux is expressed as

$$q_V = h_f(T_f - T_w)$$  (9)

The condensation heat flux can be expressed as,

$$q_C = h_f(C_f - C_w)$$  (10)

where $C_w$ is the mass concentration of saturated steam at the wall temperature $T_w$. Based on the previous studies, the Nusselt number $Nu_f$ for the average convective heat transfer coefficient in the range of $10^3 < Re_f < 10^5$ is

$$Nu_f = cRe_f^{0.6}Pr_f^m(Pr_f/Pr_w)^{0.25}$$  (11)

Zukauskas(1972) proposed $m=0.36$ and

For $S_1/S_2 < 2$ $c = 0.35(S_1/S_2)^{0.2}$  (12)

For $S_1/S_2 > 2$ $c = 0.40$  (13)

for a staggered bank. For an analogous mass transfer process, the Nusselt number and Prandtl number in the heat transfer relation Eq.(11) are simply replaced by the Sherwood number and the Schmidt number, respectively. This procedure gives

$$Sh_f = cRe_f^{0.6}Sc_f^m(Sc_f/Sc_w)^{0.25}$$  (14)

The rule gives a correct relation for the limiting situation in which the differences in temperature and concentration are vanishingly small, and it is valid for independent analogous heat and mass transfer situations as well as for a combined heat and mass transfer process.

Flue gas was treated as a mixture of $N_2$, $CO_2$, $O_2$, $CO$ and $H_2O$ and its property was estimated with special combinations of each gas property proposed by the previous studies(JSME, 1983). For example, the heat conductivity and the viscosity were estimated with the methods by Lindsay&Bromley(1950), and Wilke(1950), respectively. It is considered that a strong correlation exists between the thermal and mass diffusivities. As a first attempt, the mass diffusivity of steam in flue gas was estimated with the well-known mass diffusivity of steam in air as

$$D = D_{air} \left( \frac{\kappa}{\kappa_{air}} \right)^{11/6}$$  (15)

where $\kappa$ and $\kappa_{air}$ are the thermal diffusivities of flue gas and air, respectively. The diffusivity of steam in air can be expressed as (Fujii et. al., 1977),

$$D_{air} = 7.65 \times 10^{-5} \frac{(T + 273.15)^{11/6}}{P}$$  (16)

The previous experimental study for single stage of finned tube [Osakabe, 1998b] showed the following empirical correlation was available in the range of $2 \times 10^3 < Re_f < 5 \times 10^5$.

$$Nu_f = jRe_fPr_f^{0.33}$$  (17)

where

$$j = C_1C_3C_5\left(\frac{d + L_F}{d}\right)^{0.5}$$  (18)

$$C_1 = 0.25Re_f^{-0.35}$$  (19)

$$C_3 = 0.35 + 0.65e^{-0.25L_F/S_F}$$  (20)

$$C_5 = 0.7$$  (21)

The fin efficiency can be calculated by,

$$\eta = Y_F \left[ 0.45 \ln \left( \frac{d + L_F}{d}(Y_F - 1) + 1 \right) \right]$$  (22)

where

$$Y_F = X_F(0.7 + 0.3X_F)$$  (23)

$$X_F = \tanh(mb)$$  (24)

$$m = \left[ \frac{2h}{\lambda_f L_f} \right]^{0.5}$$  (25)

$$b = L_F + t_f / 2$$  (26)

The fin efficiency strongly depends on the heat transfer coefficient, $h$, in Eq.(25). These correlations were obtained from the single phase experiment where the heat flux is the multiple of the constant heat transfer coefficient and the temperature difference between the wall and the fluid. Although the condensation heat flux can not be expressed with the above simple relation, it is considered that the Eq.(25) with the larger heat transfer coefficient taking account of the condensation heat transfer would give an approximation. The previous study showed the fin efficiency could be evaluated with following equivalent heat transfer coefficient in the condensation region.

$$h = h_f + \beta \frac{h_f C_f - C_w}{T_f - T_w}$$  (27)

where $\beta=1$ for the usual calculation.
For an analogous mass transfer, the simple analogy between heat and mass transfer gives,

\[ \text{Sh}_f = j \text{Re}_f \text{Sc}_f^{0.33} \]  

(28)

The one-dimensional heat and mass balance calculation along the flow direction of flue gas was conducted. The steam mass concentration and the flue gas temperature at \( N+1 \)th stage can be calculated from those at \( N \)th stage as;

\[
H_2O(N) \cdot V_{wt} - \frac{qC_{AW}}{L_W} \cdot \frac{22.4}{18} = H_2O(N+1) \cdot V_{wt} - \frac{qC_{AW}}{L_W} \cdot \frac{22.4}{18} \]

(29)

\[
T_f(N+1) = T_f(N) - \frac{qV_{AW}}{C_{Pf}\rho_f(273.15 + T_f) / 273.15 \cdot V_{wt}} \]

(30)

where \( A_w \) is the heat transfer area per a stage.

It is possible that the gas temperature coincides with the dew point which is the saturation temperature corresponding to the partial pressure of steam in the flue gas. When the gas temperature decreases below the dew point, the condensation of steam in the flue gas takes place and the latent heat increases the gas temperature until the gas temperature coincides with the dew point. In this case, the energy balance gives the relation between the increase of the gas temperature, \( \Delta T_f \), and the decrease of steam concentration, \( \Delta H_2O \), as;

\[
\Delta T_f = \frac{18}{22.4} \cdot \frac{L_W}{C_{Pf}\rho_f(273.15 + T_f) / 273.15} \cdot \Delta H_2O \]

(31)

The average film thickness of condensate was calculated by the method shown in APPENDIX. As the existence of the film did not affect significantly the prediction for the present experiment, the thermal resistance of the film was neglected.

**Heat conduction in tube**

The heat conductivity for the inconel or austenite stainless steel is given with the following approximate correlation [Osakabe, 1980].

\[
\lambda_t = 13.2 + 0.013T_t \quad \text{W/(m K)}
\]

(32)

where \( T_t \) is the average temperature of tube as,

\[
T_t = \frac{T_W + T_{Wi}}{2}
\]

(33)

where \( T_w \) and \( T_{wi} \) are the outer and inner wall temperatures, respectively. The heat flux at the outer wall is,

\[
q_w = \frac{2\lambda_t(T_W - T_{wi})}{d \ln(d / d_f)}
\]

(34)

**Heat transfer in water side**

Heat transfer correlation by Dittus-Boelter taking account of the pipe inlet region is used. The inlet coefficient by McAdams(1954) is used for the modification.

\[
Nu = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \left(1 + \left(\frac{d}{L}\right)^{0.7}\right)
\]

(35)

where \( L \) is the heating length of tube.

**Pressure loss calculation**

The pressure loss per a stage of tube is,

\[
\Delta P = 2\rho_f u^2
\]

(36)

For the staggered bank of bare tube, Jacob(1938) proposed the following coefficient \( f \),

\[
f = 0.25 + \left[ \frac{0.118}{\left(\frac{S_1}{d}\right)^{0.08}} \right] \cdot \text{Re}_f^{-0.16}
\]

(37)

For the staggered bank of finned tube, the empirical correlation by ESCOA(1979) is,

\[
f = C_2C_4C_6\left(\frac{d + L_F}{d}\right)^{0.5}
\]

(38)

where \( C_2 = 0.07 + 8 \text{Re}_f^{-0.35} \)

(39)

\[
C_4 = 0.11 + \left[ \frac{0.05}{\left(\frac{S_1}{d}\right)^{0.7}} \right] \cdot \left(\frac{L_F}{S_F}\right)^{0.20}
\]

(40)

\[
C_6 = 1.1 + \left[ 1.8 - \frac{2.1e^{-0.15N_f^2}}{e^{-2.0S_2/S_1}} \right] - \left[ 0.7 - \frac{0.8e^{-0.15N_f^2}}{e^{-0.6S_2/S_1}} \right]
\]

(41)

**PARAMETRIC STUDY**

Heat transfer tubes were installed in the rectangular duct of 205x205mm to recover the latent heat in flue gas as shown in Fig.1. This duct size is approximately the same as the flue gas duct of test boiler used in this experiment. The outlet temperature of flue gas from the boiler was 280°C. The flue gas was generated with natural gas 13A at the flow rate of 15 m^3/h and the air ratio of 1.2. The flow rate and inlet temperature of feed water was 600kg/h and 20°C, respectively. The temperature of the feed water was increased from 20°C to 60°C with the heat recovery.

The various kinds of heat transfer tubes are installed in the
duct as the parametric design study. Bare tubes of 21.7, 10.5 and 5mm in outer diameter, spirally finned tubes are selected in the study. The outer diameter of finned tubes is 21.7 mm and the fin heights are 12, 8 and 3mm. The pitch of tube arrangement is larger than the tube outer diameter by 10mm considering the fabrication ability of holes at the tube sheets. The maximum number of tubes is 333 using the tube of 10.5 mm in outer diameter. The minimum number is 102 using the finned tube of 12mm in height, which is often used in conventional economizer for the sensible heat recovery. The most compact economizer is that using small bare tubes of 10.5mm in diameter. The height and the total weight of the economizer using small bare tubes are 718mm and 18.9kg, respectively. The pressure loss in the water side is slightly higher than the others but can be allowable. On the other hand, the total weight of that using the finned tubes of 12mm in height was 62.6kg. The bare tube of small diameter is preferable for the compact design of economizer for the latent heat recovery.

**Fig. 1** Boundary conditions for parametric study

**COMPACT HEAT EXCANGER EXPERIMENT**

Shown Fig.4 and 5 are a schematic of experimental apparatus. The multiple stage experiment was conducted by using a flue gas generated with the combustion of air and natural gas fuel. The flue gas from a natural gas boiler is led to the inlet plenum of the test heat exchanger using small tubes. The flue gas was released to atmosphere from the outlet plenum. The countercurrent cross-flow heat exchanger, which consist of bare tubes outer and inner diameter 10.5 and 8.5mm respectively, were designed and used for the experiment. The effective heating length of the bare tubes was 200mm.

And the small bare tubes were arranged as a bank of 10-9 rows and 40 stages. The temperature distributions of water and flue gas in the heat exchanger were measured with sheath T-type thermocouples of 0.5mm in diameter. The thermocouple signals were transferred to a data logger and analyzed. The measurement error of the temperature in this study was within ±0.1K. The pressure loss and the total amount of condensate generated in the heat exchanger were also measured. The major experimental conditions are shown in Table.3.

**Table2. Comparison of bare and finned tubes**

<table>
<thead>
<tr>
<th>Tube type</th>
<th>Bare1</th>
<th>Bare2</th>
<th>Fin1</th>
<th>Fin2</th>
<th>Fin3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter (mm)</td>
<td>10.5</td>
<td>21.7</td>
<td>21.7</td>
<td>21.7</td>
<td>21.7</td>
</tr>
<tr>
<td>Pitch (mm)</td>
<td>20.5</td>
<td>34.2</td>
<td>51.0</td>
<td>41.0</td>
<td>34.2</td>
</tr>
<tr>
<td>Stage</td>
<td>35</td>
<td>40</td>
<td>29</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>Pipe No.</td>
<td>333</td>
<td>220</td>
<td>102</td>
<td>113</td>
<td>149</td>
</tr>
<tr>
<td>Fin Pitch</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Fin Height</td>
<td>12</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>L (mm)</td>
<td>718</td>
<td>1370</td>
<td>1480</td>
<td>1030</td>
<td>923</td>
</tr>
<tr>
<td>A(m²)</td>
<td>2.25</td>
<td>3.07</td>
<td>12.4</td>
<td>8.73</td>
<td>5.04</td>
</tr>
<tr>
<td>W(kg)</td>
<td>18.9</td>
<td>46.1</td>
<td>62.6</td>
<td>51.1</td>
<td>42.5</td>
</tr>
<tr>
<td>Gas Pressure loss (mmAq)</td>
<td>11.5</td>
<td>21.6</td>
<td>19.8</td>
<td>22.1</td>
<td>22.4</td>
</tr>
<tr>
<td>Water Pressure loss (mmAq)</td>
<td>515.0</td>
<td>65.3</td>
<td>117.0</td>
<td>60.5</td>
<td>43.9</td>
</tr>
</tbody>
</table>

**Table.3 Experimental conditions**

<table>
<thead>
<tr>
<th>Air ratio</th>
<th>1.41</th>
<th>1.4</th>
<th>1.37</th>
<th>1.46</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed water (kg/h)</td>
<td>610</td>
<td>557</td>
<td>482</td>
<td>604</td>
</tr>
<tr>
<td>13A Fuel (m³/h)</td>
<td>16.1</td>
<td>15.86</td>
<td>15.9</td>
<td>16.07</td>
</tr>
<tr>
<td>Gas inlet (°C)</td>
<td>287</td>
<td>289</td>
<td>292</td>
<td>290</td>
</tr>
<tr>
<td>Water (°C)</td>
<td>21→67</td>
<td>18→71</td>
<td>19→76</td>
<td>16→67</td>
</tr>
<tr>
<td>Gas Pressure loss (mmAq)</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>23</td>
</tr>
</tbody>
</table>

**Fig. 2 Schematic of experimental apparatus for compact heat exchanger**
CONCLUSION

1. The prediction code was used on the parametric study of the heat exchanger design for the latent heat recovery. The thermal-hydraulic behavior was calculated for several kinds of heat exchangers using finned tubes or bare tubes. The calculation result indicated that the most compact heat exchanger was that using the bare tube of small diameter.

2. The compact countercurrent cross-flow heat exchanger using bare tubes of SUS304 was designed and constructed to prove its high ability. The outer and inner diameter of the bare tube were 10.5 and 8.5mm, respectively. The bare tubes were arranged in a staggered bank of 10-9 rows and
40 stages.

3. The experimental study varying the air ratio of flue gas, feed water temperature and flow rate was conducted. The experimental results for the temperature distributions of water and flue gas in the heat exchanger with bare tubes of small diameter agreed well with the prediction.

4. The proposed compact heat exchanger using small tubes was considered to be preferable for the latent heat recovery from the flue gas.

ACKNOWLEDGMENT
The authors appreciate the helpful supports by Kawasaki Thermal Engineering Co. Ltd.

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McAdams, W.H., 1954, Heat transmission, McGRAW-HILL.

APPENDIX
It is assumed that all the condensate generated at the upper stage flows on the tubes as a laminar film as shown in Fig.7. The momentum balance dominated by viscous and gravity force gives the velocity distribution at θ° from the tube top:

\[ u = \frac{(\rho_L - \rho_G)g\sin\theta}{\mu_L} \left( y\delta - y^2 \right) \]  

(42)

Integrating the above velocity profile and using the condensate mass flow rate per unit of tube length, m, yields

\[ \delta = \left[ \frac{1.5\mu_L m}{\rho_L (\rho_L - \rho_G)g\sin\theta} \right]^{1/3} \]  

(43)

The heat conductivity of film is

\[ K = \frac{\lambda_L}{\delta} = \left[ \frac{\lambda_L^3 \rho_L (\rho_L - \rho_G)g\sin\theta}{1.5\mu_L m} \right]^{1/3} \]  

(44)

The average conductivity from θ = 0° to θ = π is

\[ \overline{K} = \frac{1}{\pi} \int_0^\pi K d\theta = 0.72 \left[ \frac{\lambda_L^3 \rho_L (\rho_L - \rho_G)g}{\mu_L m} \right]^{1/3} \]  

(45)

The average heat resistance of film is defined as the inverse of the above average conductivity. The average film thickness is

\[ \overline{\delta} = \frac{\lambda_L}{\overline{K}} \]  

(46)

In the calculation, the mass flow rate, m, at a certain stage includes the condensate generated at the stage for the conservative estimation.

Fig. 7 Heat conductance of condensate film