Perception, 2000, volume 29, pages 421 – 436

DOI:10.1068/p2906

Transformation of the visual-line value in binocular vision: Stimuli on corresponding points can be seen in two different directions

Hiroshi Ono
ATR Human Information Processing Laboratories, 2-2 Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-0288, Japan; e-mail: hono@yorku.ca

Koichi Shimono
Department of Information Engineering and Logistics, Tokyo University of Mercantile Marine, Ettchujima, Tokyo 135-8533, Japan

Shinya Saida, Hiroyasu Ujike
Human Informatics Department, National Institute of Bioscience and Human Technology, 1-1 Higashi, Tsukuba 305-8566, Japan
Received 8 March 1999, in revised form 8 November 1999

Abstract. We examined Wheatstone's (1838 *Philosophical Transactions of the Royal Society of London* **128** 371 – 394) claim that images falling on retinally corresponding points can be seen in two different directions, in violation of Hering's law of identical visual direction. Our analyses showed that random-dot stereograms contain stimulus elements that are conceptually equivalent to the line stimuli in the stereogram from which Wheatstone made his claim. Our experiment demonstrated that two lines embedded in a random-dot stereogram appeared in two different directions when they stimulated retinally corresponding points, if the disparity gradient value of the lines was infinity relative to adjacent elements. To ensure that the two lines stimulated corresponding points, observers made vergence eye movements while maintaining the perception of the two lines in two different directions.

1 Introduction

Since the publication of Wheatstone's (1838) classic paper on binocular vision it has been widely accepted that stimuli which fall on non-corresponding points on the two retinae can fuse and appear in a single direction. The converse question of whether stimuli which fall on corresponding points on the two retinae can appear in two different directions, on the other hand, has yet to be fully resolved. This is therefore the question we wish to address here.

Wheatstone (1838) constructed a special stereogram, which we illustrate and discuss in the next section; after viewing it, he believed that stimuli which fall on corresponding retinal points could indeed be seen in two different directions. Soon after he made this claim, however, Brewster (1844) disagreed. Both empirical and theoretical work that followed favoured Brewster. The early attempts to reproduce Wheatstone's results failed (Brücke 1841; Hering 1862; Nagel 1861; Volkmann 1859). And, more recently, having repeated Wheatstone's and Brewster's experiments on a replica of the original apparatus, Ono and Wade (1985) found that only a few observers reported Wheatstone's perception. Moreover, had the debate taken place later, Brewster could have referred to Hering's law of identical visual direction to further his case. Such a referral would have been a powerful argument, since the law states that images falling on retinally corresponding points are seen in the same direction, though there has been a counter-argument by Helmholtz (1867/1962) that favours Wheatstone's idea (see section 2 and footnote 3).

In general, we view the question raised by Wheatstone as one concerned with the transformation of the directional values of visual lines (or local signs) into visual directions. When a 'transformation' takes place, the visual direction value of a stimulus differs from that of the visual line of the stimulus. The question, then, becomes: Is a visual direction (perceptual variable) fully determined by its visual line (physical variable) or its local sign (physiological variable)? In particular, we view the question as that of whether two visual lines of equal value can be transformed into two different visual directions, in violation of Hering's law of identical visual direction.[1] Previously, we showed that two visual lines of horizontally equal but vertically unequal values can transform into two different horizontal visual directions (Shimono et al 1998). Here we examine whether two visual lines of equal horizontal and vertical values can be transformed into two different visual directions.

In the two sections that follow we provide (a) local and global analyses of a typical random-dot stereogram, and (b) the results from an experiment designed to test Wheatstone's claim. In section 2 we draw an analogy between the elements in a typical random-dot stereogram and the line stimuli in Wheatstone's stereogram, and show how a certain perception of random-dot stereograms supports Wheatstone's claim. In section 3 we report the results from an experiment in which two lines embedded in a random-dot stereogram so as to stimulate corresponding retinal points were perceived in two different directions. To achieve this perception we constructed a stereogram which took advantage of the 'constraint' (Burt and Julesz 1980a) that only one object can be seen in a given direction. Moreover, we made certain that the two lines stimulated corresponding points by having observers make vergence eye movements, while maintaining the perception of the two lines in two different directions.

2 Visual directions produced by two retinal images stimulating corresponding points, one or both of which fused with another retinal image

The stereogram from which Wheatstone (1838) made his claim was unique among the twelve stereograms presented in his classic paper. It was unique in that it showed that stimuli which fall on corresponding points on the two retinae appear in two different directions, whereas the remainder of the stereograms showed that stimuli which fall on non-corresponding points on the two retinae fuse and appear in a single direction. The purpose of this part of the paper is (a) to illustrate Wheatstone's unique stereogram and his perception from which the claim was made, (b) to clarify his claim, (c) to show that within a typical random-dot stereogram there exist stimulus elements which are conceptually equivalent to the stimuli in Wheatstone's unique stereogram, and (d) to show that perceiving an inner disparate region of a random-dot stereogram in the centre of an outer non-disparate region affirms the idea that the stimulus elements which fall on retinally corresponding points are seen in two different directions.

[1] What constitutes a violation of Hering's law of identical visual direction should be straightforward: stimulation of corresponding points leading to two different visual directions is a violation. It is not straightforward, however, for two reasons. First, the law, as originally stated by Hering, describes the visual directions of two nonfused stimuli, and thus one might argue that it does not apply to fused images. The focus of this paper, however, is on the visual directions of fused as well as nonfused stimuli, because an analytical and empirical delineation of fused images with respect to his law is both worthwhile and important. This is true whether or not Hering himself considered fused images for the law. Second, a violation of the law of identical visual direction must be considered separately from a violation of what is sometimes called Hering's law or principle of visual direction, because what Hering stated has been modified. Thus, what constitutes a violation of the general law(s) of visual direction depends on which version of the law is being considered, and a given violation may or may not constitute a violation of the law of identical visual direction. The focus of this paper is on the law of identical visual direction and not on Hering's laws of visual direction as a whole. For recent papers addressing violations of Hering's laws of visual direction, see Erkelens and van Ee (1997a, 1997b), Erkelens et al (1996), and Shimono et al (1998). For a recent discussion of the laws of visual direction as a whole, see Banks et al (1997), Erkelens and van de Grind (1994), or van de Grind et al (1995), Howard and Rogers (1995), Mapp and Ono (1999) and Ono and Mapp (1995).

To be exact, Wheatstone's claim was that "similar pictures falling on corresponding points of the two retinae may appear as double and in different places" (page 384). This claim was based on (a) his presentation of a single thick vertical line to one eye and two lines, one thin and vertical and the other thick and tilted, to the other eye (see the top portion of the left panel of figure 1), and (b) perception of a single thick tilted line slanted in depth, intersecting a thin vertical line in a frontoparallel plane (see the lower portion of the left panel of figure 1). Perhaps, his use of the term "appear as double" was unfortunate in that some readers might interpret it to mean that two thick lines are seen (ie double of what they represent in the 3-D space). If so, it is best to mentally delete the part that reads "as double and" and skip directly to "in different places".

To understand his claim, consider the bottom point of each line as if it were a stereogram for Wheatstone–Panum's limiting case.[2] Note that the bottom end of the thick vertical line presented to the left eye (a) corresponds in size but not in retinal

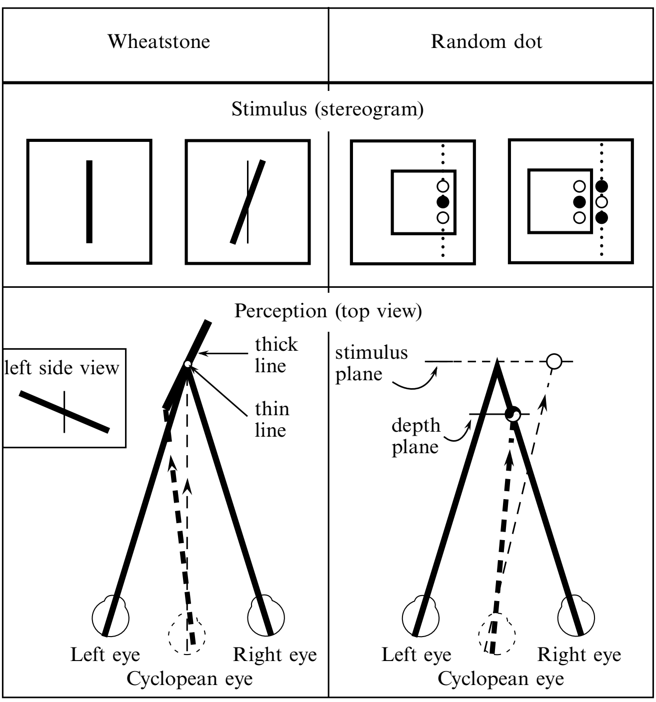

Figure 1. Wheatstone's stereogram, a schematic representation of a typical random-dot stereogram, and the perception from each stereogram when they are seen 'uncrossed'. In Wheatstone's perception, the thin line is seen in the stimulus plane, and the thick line is seen in depth. In the random-dot stereogram, the column of elements at the edge of the inner area (left field) corresponds retinally with the column of elements in the outer area (right field). The column of elements in the outer area is a 'monocular' column as there is no column of elements in the other eye that match it in pattern. The visual directions of the bottom ends of the fused lines and the fused elements are indicated by thick dashed lines, and those of the thin line and the nonfused elements by thin dashed lines. Both stereograms are drawn for illustration purpose and not for free-fusing. The slant of the thick line in the right field in Wheatstone's stereogram is exaggerated and is difficult to free-fuse with the thick line in the left field.

[2] Ono and Wade (1985) suggest that what is now called 'Panum's limiting case' be called 'Wheatstone–Panum's limiting case' following the lead of Westheimer (1976, page 62). This suggestion is made because Wheatstone (1838) was the first to study the percept resulting from presenting a single stimulus to one eye and two to the other eye.

location to the bottom end of the thick tilted line presented to the right eye, and (b) corresponds in retinal location (assuming the intersection of the visual axes is in the stimulus plane) but not in size to the bottom end of the thin vertical line presented to the right eye. In this context, Wheatstone's conclusion is that the points which correspond in size (the bottom ends of the two thick lines) fuse and are seen in a single direction, and the points which fall on corresponding retinal points (the bottom ends of the thin line in the right eye and that of the thick line in the left eye) are seen in two different directions. In the figure, the direction of the monocularly seen thin line is indicated by a thin dashed line, and the direction of the bottom end of the fused thick line is indicated by a thick dashed line.

Presented in the right panel of figure 1 is a schematic representation of stimulus elements within a typical random-dot stereogram which are conceptually equivalent to the bottom ends of the lines in Wheatstone's stereogram. Note that the 'monocular' or 'occluded' column depicted in the figure with the open circle in the middle has no column that corresponds in pattern in the other eye but does have a column (depicted with the filled circle in the middle) that stimulates retinally corresponding points in the other eye. This monocular column is analogous to the bottom part of the thin line in Wheatstone's stereogram. When the two visual axes intersect in the stimulus plane, the two columns which differ in pattern stimulate retinally corresponding points, as depicted by the dotted lines in the figure. The two columns with the filled circle in the middle correspond to the bottom parts of the two thick lines in Wheatstone's stereogram and are expected to fuse. Seeing the fused elements and the monocular elements in two different directions is equivalent to seeing the bottom part of the fused thick line and that of the thin line in Wheatstone's stereogram in two different directions. In the figure, the direction of the monocularly seen column is indicated by a thin dashed line, and the visual direction of the fused column is indicated by a thick dashed line. (Note that Brewster could have argued that the thick line or what we call the fused column is 'seen' with only one eye. What Brewster would predict is discussed in the next analysis.)

The above analysis demonstrates that Wheatstone's argument can be applied to a typical random-dot stereogram. That is, within a typical random-dot stereogram there exist stimulus elements which are conceptually equivalent to the line stimuli in Wheatstone's unique stereogram. The purpose of the analysis that follows is to demonstrate that perceiving the inner disparate region of a random-dot stereogram in the centre of the outer non-disparate region is consistent with Wheatstone's claim that stimuli which fall on retinally corresponding points can be seen in two different directions.

Random-dot stereograms provide an interesting example of Wheatstone's ideas, because they can be categorised as "stimuli on non-corresponding points fusing to appear in a single direction" and also as "stimuli on corresponding points appearing in two different directions". Which category they fall into is dependent upon how they are described. If they are described in terms of global features they fall into the first category listed above, and if they are described in terms of local features they fall into the second category. The global description is that the fused disparate inner areas of a random-dot stereogram appear in a different depth plane than the fused non-disparate outer areas, when the visual axes intersect in the stimulus plane. This description places random-dot stereograms in the same category to which all but one of Wheatstone's original stereograms belong. How the local description of random-dot stereograms puts them into Wheatstone's 'unique' category is discussed below.

Presented in the upper panel of figure 2 is a schematic representation of a typical random-dot stereogram in which four columns of dot elements have been highlighted. The figure illustrates that, when the visual axes intersect in the stimulus plane, (a) the perceived outer area excluding the monocularly seen area is composed of dot elements that correspond in pattern as well as in retinal locations, and (b) the perceived inner

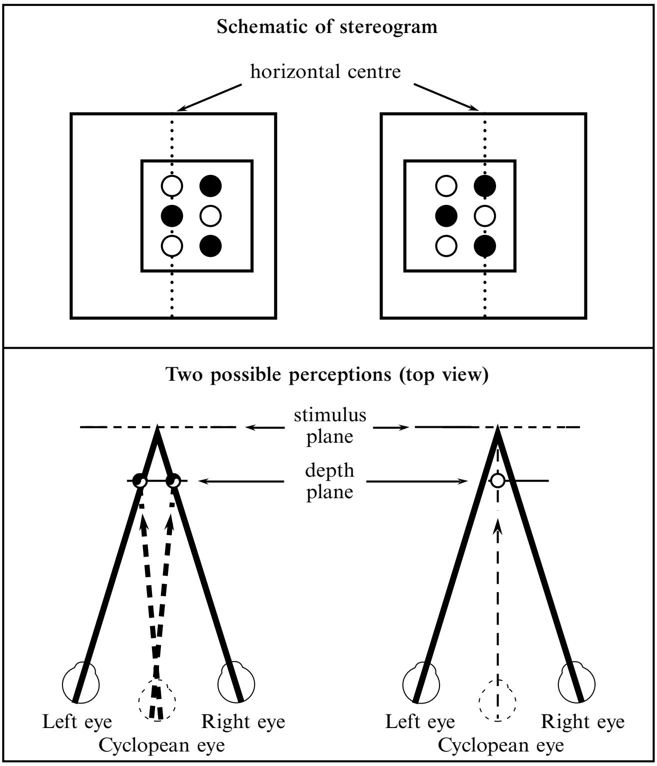

Figure 2. Schematic representation of a typical random-dot stereogram (top panel) and top views of two possible perceptions (lower panel). The columns of elements in the inner areas that occupy the centre of each field (indicated by the dotted line) fall on retinally corresponding areas when the stimulus plane is fixated. The columns of elements that correspond in pattern are also illustrated. The perception shown in the lower left panel agrees with Wheatstone's idea that "similar pictures, falling upon corresponding points of the two retinae [the columns on the dotted lines in the upper panel], may appear double and in different places [the two yinyangs in the lower panel]". The perception shown in the lower right panel agrees with the prediction from Brewster's idea that the relative direction of elements is maintained. In this case, the relative directions of the elements presented to the left eye are seen.

area is composed of dot elements that correspond in pattern but not in retinal location. Thus, in the two inner areas of the stereogram there are pairs of columns, differing in pattern, that stimulate corresponding retinal locations. One such pair, on the central meridian of each eye, is illustrated by the dotted lines at the top of the figure. Also illustrated in the figure are two columns that match in pattern with the columns on the central meridian.

Seeing the binocularly fused inner area in the centre of the binocularly fused outer area, as illustrated in the lower left panel of figure 2, supports Wheatstone's claim. That is, the two columns of dots (on the dotted lines) in the top panel of the figure, which stimulate retinally corresponding locations, are seen in two different directions by fusing with the columns corresponding in pattern. Whether one wants to call this phenomenon 'diplopia' or 'seeing double' is a question we address later, but for now we note that this perception is equivalent to the one from which Helmholtz (1867/1962) argued in favour of Wheatstone's claim. Helmholtz constructed a stereogram consisting of two parallelograms (one presented to each eye). The top and bottom edges of the parallelograms were oriented horizontally and the sides were oriented such that their upper portions produced crossed disparities with respect to their lower portions.

Moreover, the left half of each parallelogram was green, the right half was red, and the division between the two halves was parallel to the sides. Thus, near the division of colour there exist elements which stimulate corresponding points in the two retinae, but which differ in colour. When viewing this stereoscopically, he saw a trapezoidal surface inclined around a horizontal axis, the left and right halves of which were entirely green and red, respectively. This percept indicates that stimuli at corresponding points (the red and green elements near the colour division) were seen in two different directions.[3]

Not seeing the inner area of the random-dot stereogram in the centre, as illustrated in the lower right panel of figure 2, supports Brewster's claim, because the visual direction of an element agrees with the visual line to one eye. That is, the relative visual directions of the elements are the same as the relative directions of the visual lines. This perception is also predicted by suppression theory (eg Asher 1953; Kaufman 1974; Kaufman and Arditi 1976; Wolfe 1986); only the stimulus to one eye is seen, because the input to one eye suppresses the input to the other eye.

We conducted an experiment, with twelve observers, to measure the visual direction of the perceived inner area of a random-dot stereogram relative to the perceived outer area. The details of this experiment are not described here, however, to keep our exposition brief. (For conditions in which fused disparate stimuli appear in a direction which is equal to the average of the two visual lines, see eg Mansfield and Legge 1996; Ono et al 1977.) To describe the results very briefly, however, the perception illustrated in the lower left panel of figure 2 was reported by all twelve observers even when, as instructed, fixation was changed from the stimulus plane to the depth plane. There were slight individual differences in visual direction, but none of the perceptions was close to that illustrated in the lower right panel of figure 2. If one considers the local features of the stereogram, this result supports Wheatstone's claim.

When fixation was changed from the stimulus plane to the depth plane, the areas of the stereogram that corresponded retinally shifted from the outer area to the inner area. Accordingly, while the fixation is on the depth plane, the argument concerning retinal images on corresponding points appearing in two different directions must be made for columns in the outer area. A more critical point, however, is whether there is a fixation error or not, because stimulation of retinally corresponding points depends on accurate fixation. This point is addressed in section 3.

Independently of whether fixation is on the stimulus plane or the depth plane, seeing the inner area centrally with respect to the outer area represents stimuli at non-corresponding points fusing and appearing in a single direction. Moreover, this percept can be thought of as resulting from the transformation of two different visual-line values, associated with elements in the two eyes (or that of the inner or outer areas as a whole), into a single visual direction, the value of which equals the average of the two visual line values. A consequence of this is that images which stimulate corresponding points are seen in two different directions. Whether this perception violates Hering's law of identical visual direction depends on the domain and the range of the law. If the law is intended to describe any stimulation of corresponding points, it is a violation. If the law is restricted to nonfused stimuli as in Hering's original demonstration, however, it is not. (See footnote 1 for further discussion.)

Whether Hering's law is violated or not, the results imply that stimuli falling on corresponding points are more likely to be seen in different directions than in an identical direction. Seeing these points in an identical direction is restricted to the case in which they are on the plane of fixation (simulated or real), or on a horopter, to be exact. Whenever there are two or more perceived planes (or a perceived surface that is inclined as that produced by Helmholtz's stereogram described above), there will be stimuli

[3] For an illustration of Helmholtz's stereogram, see page 450 in Helmholtz (1867/1962).

falling on corresponding points that are seen in different directions. To go one step further, if convergence is not on a perceived plane (eg convergence is between the stimulus plane and the depth plane in figure 2) then all pairs of stimuli acting on corresponding points are seen in different directions, consistent with Wheatstone's idea.

Two details should be noted before moving on to section 3. First, one should not conclude that a perception compatible with Brewster's is impossible, for it does occur when a disk and an annulus with large retinal disparity are used as stimuli (Ono et al 1977). Whether it occurs with a random-dot stereogram, however, is not yet known. Second, what we call Wheatstone's perception is not typically referred to as diplopia. Usually, diplopia or 'seeing double' refers to unfused binocular stimuli which appear in two different directions. A perception closer to what is usually called diplopia and which also conforms to Wheatstone's claim is examined in section 3, where we designed a stimulus in which two lines embedded in a random-dot stereogram, which fall on corresponding points without fusing, appear in two different directions.

3 Visual directions produced by two retinal images stimulating corresponding points, neither of which fused with another retinal image

Our analyses of the random-dot stereogram, presented in section 2, suggest that elements which stimulate retinally corresponding points can be seen in two different directions when they fuse with their 'matching' elements in the stereogram. In this section we present the results from an experiment designed to show that lines embedded in a random-dot stereogram, in such a way that they fall on corresponding points in the two retinae but do not fuse, can also be seen in two different directions. To assess the implications of our results it is crucial to understand, first, how the stereogram was constructed; second, what observers saw when they viewed the stereogram; and, third, how we ensured that the target lines embedded in the stereogram stimulated corresponding points on the two retinae.

First, two sets of stereograms were made, a stationary set and a dynamic set. Within each set, the inner area of one stereogram had crossed disparity relative to the outer area and the inner area of the other stereogram had uncrossed disparity. Each of the four stereograms was designed to take advantage of the 'constraint' (Burt and Julesz 1980a) that only one object can be seen in a given direction. [Two objects in the same visual direction have a disparity gradient value of infinity, and a disparity gradient larger than unity is expected to produce diplopia (Burt and Julesz 1980a, 1980b).] Specifically, a black fusional line (constraining stimulus) was presented in the centre of the inner areas (one to each eye) of each stereogram so that when fused it appeared straight-ahead. See the left panel of figure 3. Also, in the inner area of each field, one of two differently coloured target lines was presented at the centre with respect to the outer area so that when fixation was at the stimulus plane the two target lines (one to each eye) stimulated corresponding retinal points. Thus, if the images of the two coloured target lines were to fuse, the fused line would appear in the same direction as the fused black line (ie straight-ahead). According to the constraint mentioned above, however, this is not possible and, therefore, one of the pairs cannot fuse. Given that the two target lines differed in colour, it is more likely that the pair of black lines would fuse, because two differently coloured lines will not fuse unless they are flashed for less than 100 ms (eg see Ono et al 1971).

Second, as expected, the black fusional lines in each of the four stereograms fused and served as a constraining stimulus. A black line appeared single and straight-ahead, and the two coloured lines appeared in two different directions. An illustration of the expected percept for one of the stereograms is presented in the right panel of figure 3. For the results from an experiment to estimate the proportion of the population that would experience the required perception, see the appendix.

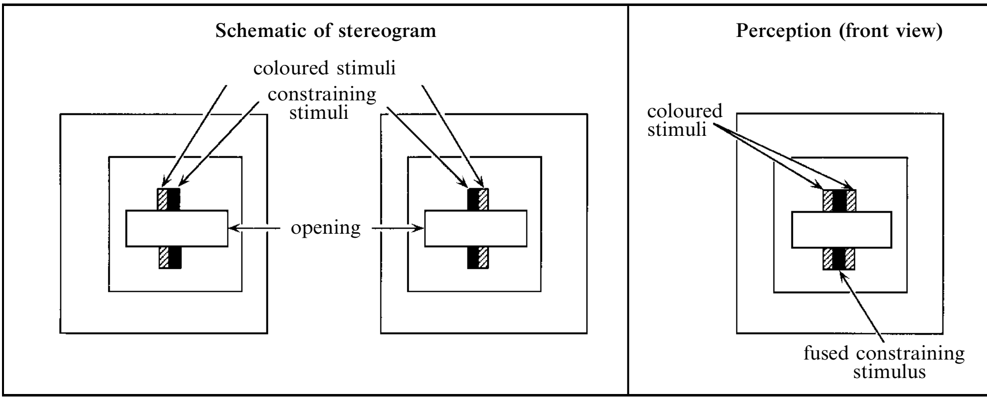

Figure 3. Schematic representation of one of the random-dot stereograms used and the perception (front view). The 'constraining' stimulus is in the centre of the inner area in each field of the stereogram; the coloured lines are in the centre with respect to the outer area and fall on corresponding points when the stimulus plane is fixated. The 'opening' through which the moving stimulus is viewed appears in the centre of the binocularly fused inner area. The perception (the right panel) consists of seeing one coloured line on the left and the other coloured line on the right of the fused constraining stimulus. (Note that the additional black lines described in the text and presented in the stereograms posted on the *Perception* web site are not shown here so as to simplify the figure.)

Third, to ensure that the coloured target lines stimulated retinally corresponding points, the observers' binocular eye movements were measured under the following conditions. In the stationary-stereogram condition, observers tracked a pursuit stimulus that appeared to move back and forth in the median plane. The range of each eye's motion was such that the monocular images of the two coloured lines fell on corresponding points for at least one moment during each half-cycle. In the dynamic-stereogram condition, the entire stimulus field presented to each eye moved leftward and rightward in counterphase. Because of the lag in tracking such a stimulus, the monocular images of the target lines fell on corresponding points before and after the time at which the movement of the fields changed direction.

3.1 *Methods*

3.1.1 *Observers.* Two adult males (TO and YT, 20 and 21 years old, respectively), both inexperienced in eye-movement and psychophysical experiments, served as observers. Both reported normal visual acuity and neither had difficulty in perceiving the apparent depth in a stereoscope. Three authors of this paper also participated in the experiment and all three repeatedly obtained results consistent with those reported in the results section. Only the eye-movement results of the two inexperienced observers are reported.

3.1.2 *Apparatus and stimuli.* The apparatus consisted of a mirror stereoscope constructed from two colour monitors (Iiyama MT8617E), each connected to a computer (Apple Power Macintosh 8500). The centres of the monitors were set at eye level at an optical distance of 57 cm.

In the stationary stereograms, each field subtended 359×300 elements and the inner area subtended 143×120 elements, with a disparity of 9.2 min of arc. The inner area had an 'opening' of 8.20 deg $\times$ 0.61 deg in which a pursuit stimulus moved back and forth. Each element subtended 4.6 min of arc. The two coloured test targets and the constraining stimuli were embedded above and below the opening in the inner area. In addition, black lines flanked the coloured test targets on the side opposite the constraining stimuli. These additional black lines are not depicted in figure 3 so as to simplify the figure. The test targets consisted of a red and a green line (0.08 deg $\times$ 2.10 deg each) and were positioned in the centre with respect to the outer area. The colour of the line on the left side above the opening was the same as the colour of the line on the right side below the opening. The constraining stimuli, which consisted of a pair of black lines (0.08 deg $\times$ 2.10 deg each), were presented in the centre of the inner area and served to prevent the coloured lines from fusing. The additional black lines (0.08 deg $\times$ 2.10 deg each) were embedded above and below the opening in the inner area, as described above, and served to make the percept of the coloured lines more stable. The eye-movement target, a white bar (18.5 min of arc $\times$ 4.6 min of arc) in each opening, moved laterally and sinusoidally at the centre of the opening, and the fused target appeared to move toward and away from the observer.

The overall size of the dynamic stereograms was smaller than that of the stationary ones to allow each field to move laterally on the screen. Each field of the dynamic stereograms subtended 49 $\times$ 40 elements and the inner area subtended 29 $\times$ 24 elements, with a disparity of 18.4 min of arc. The inner area had an 'opening' of 3.20 deg $\times$ 0.61 deg (which was not necessary for the purpose of the experiment other than to make it similar to the stationary one). Each element subtended 9.2 min of arc. The coloured test targets, the constraining stimuli, and the additional black lines were embedded above the opening in the inner area. Those below the opening were removed after an observer in a preliminary experiment reported difficulty in paying attention to the coloured stimuli below the opening. The bottom of the fused constraining stimulus served as a fixation point. As in the stationary stereograms, the test targets consisted of a red and a green line positioned in the centre with respect to the outer area, but they were wider and shorter (0.15 deg $\times$ 0.92 deg each). The constraining stimuli, a pair of black lines (0.15 deg $\times$ 0.92 deg each), were presented in the centre of the inner area. The additional black lines (0.15 deg $\times$ 0.92 deg each) flanked the coloured test targets on the side opposite the constraining stimuli, as in the stationary stereograms. The whole stimulus for each eye moved laterally and sinusoidally in opposite directions on the screen. Given Regan et al's (1986) finding, we expected that the fused stimulus would not appear to move toward and away from the observer, but three out of five observers, including one inexperienced observer (YT), perceived some motion. We have not investigated the reason for this unexpected result, because whether or not motion was perceived was not directly relevant to the purpose of our experiment.

Both the stationary and dynamic stereograms described above are available on the *Perception* website (www.perceptionweb.com/perc0400/ono.html) and archived on the annual CD-ROM provided with issue 12 of the journal All stereograms posted on the site contain the additional black lines described above but not shown in figure 3.

Binocular eye movements were recorded with a photoelectric apparatus designed and constructed by the third author and an analog data recorder (Sony Instrumentation KS-609). The system was capable of measuring horizontal eye movements linearly in the range of 6 deg with a resolution of 2 min of arc. Before and after each session, the system was calibrated by having observers fixate two stimuli on computer screens separated horizontally by 4.0 deg.

3.1.3 *Procedure.* To allow for individual differences in the ability to maintain the percept of two coloured lines appearing in two different directions, we first approximated for each observer the maximum amplitude of eye movement that could be made while still maintaining the percept. This was achieved by varying the amplitude and the frequency of the motion of the eye-movement targets in the stationary stereograms and by varying those of the motion of the whole stimuli for the dynamic ones. During this manipulation observers reported the disappearance of one of the coloured lines by pressing a button. We performed this procedure because occasionally one or both of the coloured lines disappeared. (This binocular rivalry is not necessarily between the two coloured lines; it can be between a coloured line and pattern in the random-dot stimulus, depending on the eye position.) For the stationary stereogram, the optimal amplitude was 18.5 min of arc and the optimal frequency was 0.2 Hz for both observers. For the dynamic stereogram, the optimal amplitudes were 2.30 deg and 0.92 deg for TO and YT, respectively, and the optimal frequency was 0.5 Hz for both observers.

Eye-movement data were collected in the following four conditions: stereogram type (stationary or dynamic) $\times$ disparity sign (crossed or uncrossed). For the stationary condition, the stimulus was presented for 3 cycles of movement; eleven times in each disparity condition for TO and six times in each condition for YT. For the dynamic condition, the stimulus was presented for 15 cycles of movement; twice in each disparity condition for TO and once in each disparity condition for YT. Because TO blinked frequently, the stimulus was presented more often for TO than for YT. In each session, the observers were asked to report, by pressing a button, when one of the coloured lines disappeared. Neither observer, however, reported the disappearance during the experiment.

Before the stimulus presentation, but after calibration, nonius stimuli, consisting of a pair of 'binocular' squares (9.2 min of arc $\times$ 9.2 min of arc) and a pair of monocular vertical lines (9.2 min of arc $\times$ 69.2 min of arc), one above the square and the other below, were presented on blank screens. When the nonius stimuli appeared aligned, the stereogram was presented. For the eye position indicated by the alignment, the two coloured lines stimulated corresponding points.

3.2 *Results and discussion*

For the two sets of stereograms, both inexperienced observers reported that the two coloured lines appeared in two different directions throughout the eye-movement cycles. With the stationary stereograms, the perception consisted of a stationary random-dot pattern with the inner area in front or behind the outer area, and the white bar moving toward and away from the observer in the opening. The two different coloured lines were seen in two different locations on the inner area. With the dynamic stereograms, the perception was basically the same, except YT perceived a slight forward and backward movement of the whole stimulus. Because the two lines were seen in two different directions, the crucial question became whether or not the coloured lines stimulated corresponding retinal points at some time during the eye-movement cycle.

To answer this question the observer's binocular eye movements were analysed as follows. The eye-movement records without eye blinks were filtered with a cutoff frequency of 20 Hz, and then sampled and digitised every 10 ms with the use of Maclab/8. Then the difference between the left and right eye positions was computed and used as an index of 'vergence' movement.

For the results from the stationary stereograms, usable eye-movement cycles were averaged to increase the signal-to-noise ratio. The averaged vergence movements for the two stereograms from the two observers are shown in figure 4 along with the exact number of eye-movement cycles used for each trace. In the figure, the value of zero on the ordinate represents the point at which the two coloured lines, seen in two different directions, stimulated retinally corresponding points. The averaged eye-movement traces show that tracking vergence followed the moving target relatively well, although TO's trace from one condition shows that the eye movement preceded and overshot the moving target. For both observers, the magnitude of vergence movements was more than twice the disparity value of the stereogram (9.2 min of arc), and we conclude that the two coloured lines stimulated retinally corresponding points at least once in each cycle.

For the results from the dynamic stereograms, averaging was not necessary because eye-movement amplitudes were much larger. In addition to computing the vergence eye movement, a 'deviation' score was computed. This score consists of the difference between the actual vergence position and the vergence position required for the two

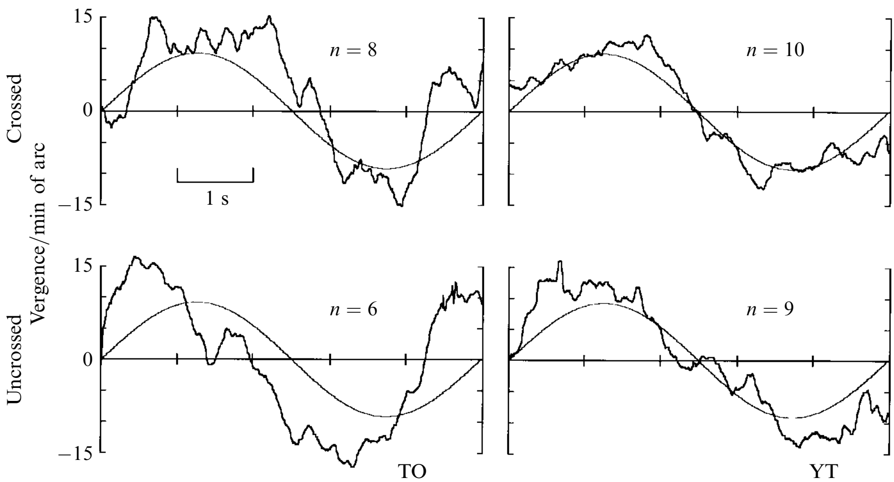

Figure 4. Averaged vergence eye-movement traces and target-movement traces with the static stereogram from the two inexperienced observers.

coloured lines and the dots in the outer area to stimulate corresponding points. A sample of the deviation trace is shown in figure 5 along with a trace of the right-eye and the left-eye movements and a trace of the vergence eye movement. The deviation traces obtained with the two stereograms from the two observers are shown in figure 6. In figures 5 and 6, the value of zero on the ordinate of the deviation score represents the point at which the two coloured lines, seen in two different directions, stimulated retinally corresponding points. The deviation traces clearly show that the eyes deviated

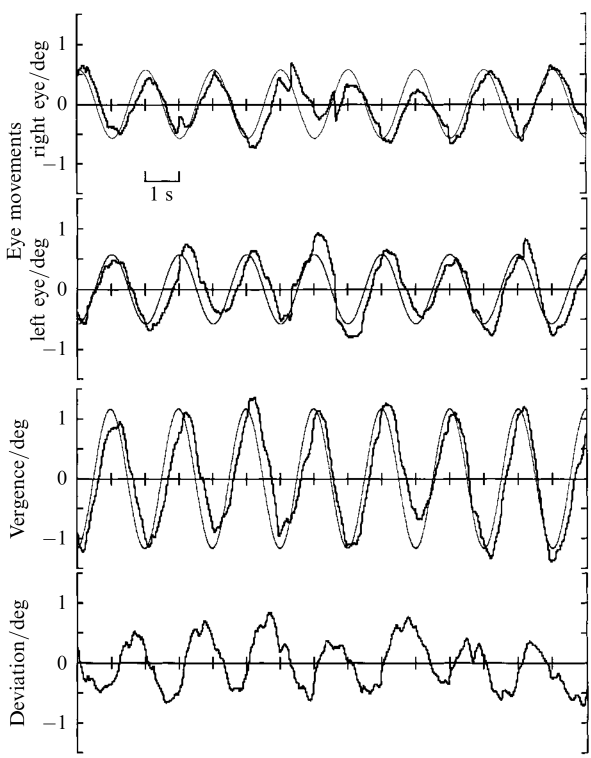

Figure 5. Sampled eye-movement traces, their vergence eye-movement trace, and its deviation trace. Sample taken from the record of TO's eye movement with the 'crossed' dynamic stereogram.

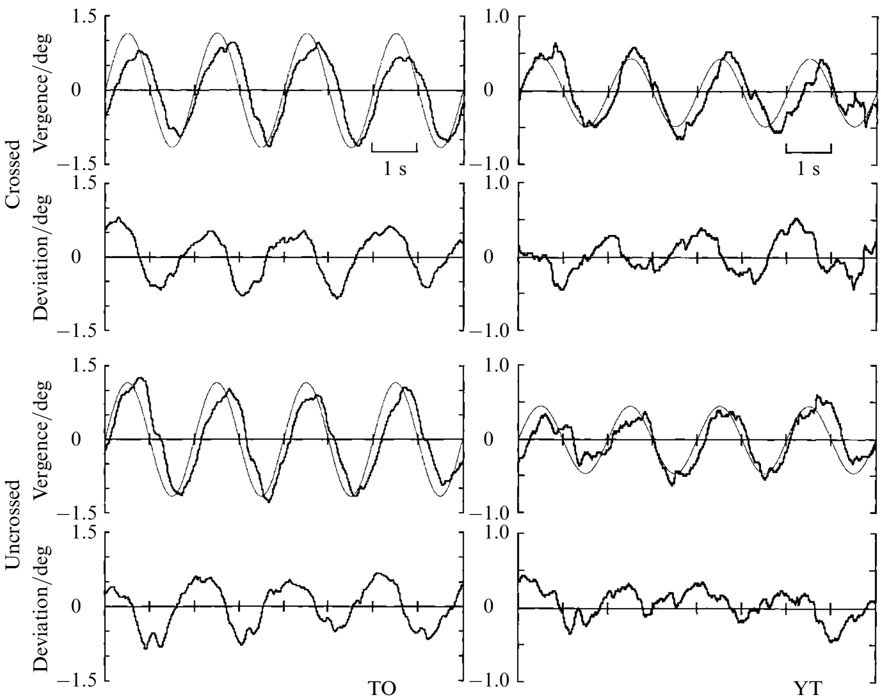

Figure 6. Vergence traces superimposed on position traces, and the deviation traces with the dynamic stereogram from the two inexperienced observers.

from the stimulus position, and, more importantly, that they moved in such a way that the two coloured lines stimulated corresponding points.

The traces shown in figures 4, 5, and 6 taken together provide very strong support for Wheatstone's claim, because it is almost certain that the two coloured lines which were seen in two different directions stimulated corresponding points. The reason for the qualification "almost certain" is that the eye-movement recording technique we used measured the magnitude of the eye movements accurately, but not the absolute position of the eyes. Accuracy in measuring the eye position depends on the accuracy of convergence (or fixation) during the calibration trial. If that accuracy was high, the coloured lines stimulated corresponding points when an eye-movement trace crossed the zero point indicated on the ordinate. However, even if there was a convergence error during the calibration as large as 18.4 min of arc (a value twice the disparity, which is very unlikely), we can conclude that the lines stimulated corresponding points.

Three more comments regarding stimulation of corresponding points are offered below. The first is concerned with a stereogram in a stereoscope, the second with a stimulus on an empirically defined vertical horopter, and the third with seeing two coloured lines in two different directions.

First, the loci of stimuli in space that stimulate corresponding points are limited to a Vieth–Müller circle and a vertical horopter located in the median plane. This limitation, however, does not necessarily exist in the 'virtual reality' of stereoscopic space. If the mirrors are arranged so that each field is presented normal to the visual axis, as ours were, all the elements in the stereogram stimulate corresponding points when binocular fixation is at the stimulus plane. Thus, a stereoscope allows a test of Hering's law of identical visual direction for visual lines that cannot be tested in 'real' space.

Second, we can calculate the eye position required to stimulate corresponding points defined by an empirical vertical horopter. A stimulus on this horopter does not stimulate the retinally vertical meridian but rather the meridian that is tilted approximately 1° with the top toward the temporal side in each eye (Cogan 1979; Helmholtz 1867/1962; Nakamizo et al 1999; Nakayama 1977). For the stationary stereograms, the ends of the coloured lines very near the top or the bottom of the opening stimulated empirically defined corresponding points when the vergence position was 0.7 min of arc more or less than the vergence position at the stimulus plane, depending on whether the eyes were diverging or converging. The ends of the lines at the top or the bottom of the inner areas stimulated corresponding points when the vergence position was 4.9 min of arc more or less than the vergence position at the stimulus plane. Because these values are considerably smaller than the vergence movement of about 18 min of arc, we can conclude that the coloured lines stimulated empirically defined corresponding points and were seen in two different directions. For the dynamic stereograms, the bottom ends of the coloured lines stimulated retinally and empirically defined corresponding points when the eyes reached the stimulus plane, because the fixation was on the bottom of the fused constraining stimulus. The top ends of the lines on the inner areas stimulated corresponding points when the vergence position was 1.3 min of arc more or less than the vergence position at the stimulus plane, depending on whether the eyes were diverging or converging. Because this value is smaller than the deviation score, the same conclusion applies here to the stationary stereograms. In sum, we can conclude that a stimulus that stimulates corresponding points, whether defined empirically or defined retinally, can be seen in two different directions.

Third, the stability of visual direction during the eye movement, including the moment in which corresponding points were stimulated by the two coloured lines, was maintained for our two inexperienced and three experienced observers. This result may suggest that visual-line values and eye positions are monitored continuously, since both were changing continuously. This suggestion, however, would imply placing an unnecessary burden on the visual system when it is dealing with a complex stimulus. A more parsimonious way would be to achieve a perception of direction from the initial information about the visual lines and eye positions, and then maintain that percept within a certain time period or until the next 'sampling' of the information that contradicts the perception. This idea suggests that the perception of the two coloured lines, when corresponding points are stimulated, is partly dependent on 'lag time' or hysteresis'; since the stability of the required perception is not the most 'robust' or reliable one when the eyes are fixed, as indicated in the data summarised in the appendix. Nonetheless, the stability of seeing two lines in two different directions with the eye movement shown by our observers is sufficient to provide strong support for Wheatstone's claim that "similar pictures falling on corresponding points of the two retinae may appear ... in different places" (page 384, our italics). Moreover, the demonstrated transformation of visual-line values has an important methodological implication for monitoring eye position with nonius stimuli. This is because the nonius method assumes that the value of a visual line is not transformed. This issue is addressed in Ono and Mapp (1995), Rogers and Bradshaw (1999), and Shimono et al (1998).

4 General discussion

The analyses and experimental results presented in this study confirmed Wheatstone's idea that visual-line values which stimulate retinally corresponding points are labile, just as are visual-line values which stimulate disparate retinal points. Confirmation came in section 2 from analyses of the perception of a random-dot stereogram and in section 3 from experimental results obtained with modified random-dot stereograms.

The analyses in section 2 indicate that (a) monocular columns of elements in a random-dot stereogram and the columns of elements that stimulate retinally corresponding points in the other eye appear in different directions, and (b) binocular columns of elements in a random-dot stereogram that stimulate corresponding points in the two retinae appear in two different directions by fusing with the columns of elements which stimulate non-corresponding points. Although the initial (and more recent) attempts to confirm Wheatstone's claim (Brücke 1841; Hering 1862; Nagel 1861; Ono and Wade 1985; Volkmann 1859) were not successful, the perception that supports his claim can be easily obtained by using a random-dot stereogram. The initial difficulty was probably due to (a) the large retinal disparity of the thick lines which resulted in only a few observers being able to fuse the entire length of the lines (Helmholtz 1867/1962) and (b) the use of a stereogram with line drawings instead of one with a larger number of fusible elements as in a random-dot stereogram.

The results from the experiment reported in section 3 also support Wheatstone's claim. Unlike the perception of Wheatstone's stereogram, or those analysed in section 2, stimulus elements which fell on retinally corresponding points appeared in two different directions without fusing with other elements. To obtain this perception, however, by simply inserting a line in each field of a random-dot stereogram would not be successful. Without the 'constraining' stimuli, the lines would fuse and be seen at the stimulus plane in agreement with Hering's law of identical visual direction (Ono 1991). When the inner area has an uncrossed disparity with respect to the outer area, this fused single line would be seen 'floating' on the stimulus plane. When the inner area has a crossed disparity with respect to the outer area, this fused single line would be seen again at the stimulus plane, but through a 'hole' in the inner area.

The results reported in section 3 violate both Brewster's claim and Hering's law of identical visual direction. Clearly, the universality of the claim and the law have been falsified. These results, however, in no way argue against Brewster's observation or the validity of Hering's law of identical visual direction in a particular stimulus situation. As mentioned previously, Brewster's observation has been confirmed for stimuli with large retinal disparity (Ono et al 1977), and a perception equivalent to that in Hering's (1879/1942) classical demonstration (the two nonfused images of the tree top and the chimney appearing in the same direction when they stimulate corresponding points), undoubtedly, has been observed by many visual scientists. What is needed now are more experimental determinations of the stimulus conditions to which Brewster's observation or Hering's law apply, and of those to which they do not. To reiterate the suggestion made elsewhere (Ono et al 1977; Ono and Wade 1985), the empirical issue of these perceptions is not an 'either or' proposition; the transformation of visual-line values, or the lack thereof, depends on the experimental conditions.

Acknowledgements. This research was supported by Grant A0296 from the Natural Sciences and Engineering Research Council of Canada, a grant from ATR Human Information Processing Research Laboratories (Kyoto, Japan), and a grant from the Centre for Research in Earth and Space Technology (Toronto). The experiment was conducted at the National Institute of Bioscience and Human Technology (Japan), and most of the writing (1995) and the revision (1997) were done while the first author was a visiting researcher at ATR Human Information Processing Research Laboratories (Kyoto, Japan). Masahiro Suzuki made the stereograms for the 'double-blind' experiment described in the appendix, and Mirona Gheorghiu and Simon Sherry conducted the experiment. We wish to thank them for their help. In addition, we wish to thank Drs Robert S Allison, Esther González, Ian P Howard, Brian Rogers, Martin J Steinbach, and Nicholas J Wade for their helpful comments on an earlier version of this paper, and Alistair P Mapp and Linda Lillakas for their editorial help.

References

Asher H, 1953 "Suppression theory of binocular vision" *British Journal of Ophthalmology* **37** 37 – 49

Banks M S, Ee R van, Backus B T, 1997 "The computation of binocular visual direction: A re-examination of Mansfield and Legge (1996)" *Vision Research* **37** 1605 – 1610

Brewster D, 1844 "On the law of visible position in single and binocular vision and on the representation of solid figures by the union of dissimilar plane pictures on the retina" *Transactions of the Royal Society of Edinburgh* **15** 349 – 368

Brücke E, 1841 "Über die stereoscopischen Erscheinungen und Wheatstone's Angriff auf die Lehre von den identischen Stellen der Netzhäute" *Archiv für Anatomie, Physiologie und Wissenschaftliche Medizin* **8** 459 – 476

Burt P, Julesz B, 1980a "Modifications of the classical notion of Panum's fusional area" *Perception* **9** 671 – 682
Burt P, Julesz B, 1980b "A disparity gradient limit for binocular fusion" *Science* **208** 615 – 617
Cogan A I, 1979 "The relationship between the apparent vertical and the vertical horopter" *Vision Research* **19** 655 – 665
Erkelens C J, Ee R van, 1997a "Capture of visual direction: An unexpected phenomenon in binocular vision" *Vision Research* **37** 1193 – 1196
Erkelens C J, Ee R van, 1997b "Capture of visual direction of monocular object by adjacent binocular objects" *Vision Research* **37** 1735 – 1745
Erkelens C J, Grind W A van de, 1994 "Binocular visual direction" *Vision Research* **34** 2963 – 2969
Erkelens C J, Muijs A J M, Ee R van, 1996 "Binocular alignment in different depth planes" *Vision Research* **36** 2141 – 2147
Grind W A van de, Erkelens C J, Laan A C, 1995 "Binocular correspondence and visual direction" *Perception* **24** 215 – 235
Helmholtz H von, 1867/1962 *Treatise on Physiological Optics* volume 3 (New York: Dover, 1962); English translation by J P C Southall for the Optical Society of America (1925) from the 3rd German edition of *Handbuch der physiologischen Optik* (first published in 1867, Leipzig: Voss)
Hering E, 1862 *Beiträge zur Physiologie* volume 2 (Leipzig: Engelmann)
Hering E, 1879/1942 *Spatial Sense and Movement of the Eye*; English translation by C A Raddle (Baltimore, MD: American Academy of Optometry, 1942) of "Der Raumsinn und die Bewegung des Auges", in *Handbuch der Physiologie* Ed. L Hermann, Band 3, Teil 1 (Leipzig: Vogel, 1879)
Howard I, Rogers B, 1995 *Binocular Vision and Stereopsis* (New York: Oxford University Press)
Kaufman L, 1974 *Sight and Mind* (New York: Oxford University Press)
Kaufman L, Arditi A, 1976 "The fusion illusion" *Vision Research* **16** 535 – 543
Mansfield J S, Legge G E, 1996 "The binocular computation of visual direction" *Vision Research* **36** 27 – 41
Mapp A P, Ono H, 1999 "Wondering about the wandering cyclopean eye" *Vision Research* **39** 2381 – 2386
Nagel A, 1861 *Das Sehen mit zwei Augen und die Lehre von den identischen Netzhautstellen* (Leipzig: Winter)
Nakamizo S, Ono H, Ujike H, 1999 "Subjective staircase: A multiple wallpaper illusion" *Perception & Psychophysics* **61** 13 – 22
Nakayama K, 1977 "Geometric and physiological aspects of depth perception" *Proceedings of the Society of Photo-Optical Instrument Engineers* **120** 2 – 9
Ono H, 1991 "Binocular visual directions of an object when seen as single or double", in *Binocular Vision* Ed. D Regan, volume 9 of *Vision and Visual Dysfunction* Ed. J R Cronly-Dillon (London: MacMillan) pp 1 – 18
Ono H, Angus R, Gregor P, 1977 "Binocular single vision achieved by fusion and suppression" *Perception & Psychophysics* **21** 513 – 521
Ono H, Komoda M, Mueller E R, 1971 "Intermittent stimulation of binocular disparate colors and central color fusion" *Perception & Psychophysics* **9** 343 – 347
Ono H, Mapp A P, 1995 "A restatement and modification of Wells – Hering's laws of visual direction" *Perception* **24** 237 – 252
Ono H, Wade N, 1985 "Resolving discrepant results of the Wheatstone experiment" *Psychological Research* **47** 135 – 142
Regan D, Erkelens C J, Collewijn H, 1986 "Necessary conditions for the perception of motion in depth" *Investigative Ophthalmology & Visual Science* **27** 584 – 597
Rogers B J, Bradshaw M F, 1999 "Disparity minimisation, cyclovergence, and the validity of nonius lines as a technique for measuring torsional alignment" *Perception* **28** 127 – 141
Shimono K, Ono H, Saida S, Mapp A P, 1998 "Methodological caveats for monitoring eye position with Nonius stimuli" *Vision Research* **38** 591 – 600
Tyler C, 1991 "The horopter and binocular fusion", in *Binocular Vision* Ed. D Regan, volume 9 of *Vision and Visual Dysfunction* Ed. J R Cronly-Dillon (London: MacMillan) pp 19 – 37
Volkmann A W, 1859 "Die stereoskopischen Erscheinungen in ihrer Beziehung zu der Lehre von den identischen Netzhautstellen" *Archiv für Ophthalmologie* **5** 1 – 100
Westheimer G, 1976 "Oculomotor control: The vergence system", in *Eye Movement and Psychological Processes* Eds R A Monty, J W Senders (Hillsdale, NJ: Lawrence Elbaum Associates) pp 55 – 64
Wheatstone C, 1838 "Contribution to the physiology of vision—Part the first. On some remarkable, and hitherto unobserved, phenomena of the binocular vision" *Philosophical Transactions of the Royal Society of London* **128** 371 – 394
Wolfe J M, 1986 "Stereopsis and binocular rivalry" *Psychological Review* **93** 269 – 282

APPENDIX

Perception of seeing the lines in two different directions

To estimate the proportion of the population that would experience the perception required for section 3, we performed a 'double-blind' experiment. Two 'naïve' experimenters presented eight stereograms to eighteen 'naïve' observers. All were undergraduate students, who were naïve as to the purpose of the experiment and as to the expected results. The two experimenters were paid for their service and the observers received 'credits' toward an introductory psychology course. Two of the stereograms are described and results from seventeen observers (one was stereoblind) are reported below. (A more detailed description can be obtained by writing to the first author.)

The stereograms were very much like those used in section 3 and contained random dots, test targets (a dashed line and a thin line), a 'constraining' stimulus, and a 'window'. In addition, there were two fixation stimuli and nonius stimuli. One fixation point (a green dot) was presented in the outer area 15 min of arc above the top of the constraining stimulus, and the other was presented in the inner area 15 min of arc below the top of the constraining stimulus. (The visual angle subtended by the stereogram was in between the one used in the 'stationary' condition and the 'dynamic' condition of section 3.) The stereogram filled the screen (16 deg $\times$ 12 deg) and the inner area was 7.6 deg $\times$ 6.4 deg. The nonius stimuli consisted of two red LEDs. They appeared aligned (one above the other) or nearly so when the outer area was fixated, and nonaligned (horizontally separated) when the inner area was fixated. The nonius stimuli were used to train the observers and to check how well they were following instructions.

The stereograms were presented at the optical distance of 75 cm with a stereoscope provided with beam splitters, which allowed independent presentation of the nonius stimuli and the stimulus elements in the stereograms. In any given viewing period, each stereogram was presented with the inner area having either crossed or uncrossed disparities (15 min of arc) relative to the outer area. It was viewed in one of two different ways: alternating fixations back and forth at a 'comfortable' rate between the two fixation points (the alternating-fixation condition) or fixating the fixation point on the background (the fixation condition). Each viewing lasted 30 s, during which time the experimenter asked the observer questions about his/her percept.

In the alternating-fixation condition, seventeen (100%) observers reported fusion (seeing 'correct' depth and seeing a single thick line, ie the constraining stimulus) when the inner area was both crossed and uncrossed. Only the thick line above the 'window' was considered here, because observers reported that they could not 'see' the line below the 'window' (its top was over 3 deg away from the fixation points). Of these seventeen, eight (45%) observers saw the two test lines 'all the time'; and seven (41%) saw it 'most of the time' when the inner area was crossed. Nine (53%) observers saw them 'all the time'; and seven (41%) 'most of the time' when it was uncrossed.

In the fixation condition, fifteen (88%) observers reported fusion when the inner area was crossed and sixteen (94%) when it was uncrossed. Of the fifteen, six (40%) observers saw two test lines 'all the time' and four (27%) 'most of the time' when the inner area was crossed. Of the sixteen, seven (44%) observers saw them 'all the time'; and six (38%) 'most of the time' when it was uncrossed.

The sums of the percentages of 'all of the time' and of 'most of the time' are 88%, 94%, 67%, and 81%, respectively, for the conditions described above. The point being made with these percentages is that many observers saw the two test targets and that the perception is not limited to the observers who participated in section 3.