

2.

$$f = x + y^2 + z$$

$$g = x^2 + 3y^2 + 2z^2 - 1 = 0 \quad \text{とある}$$

$$\nabla f + \lambda \nabla g = 0 \quad \text{より}$$

$$\begin{pmatrix} 1 \\ 2y \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2x \\ 6y \\ 4z \end{pmatrix} = 0 \quad \therefore \begin{cases} 2\lambda x + 1 = 0 \\ y(6\lambda + 2) = 0 \\ 4\lambda z + 1 = 0 \end{cases}$$

$$\therefore y = 0, \quad x = -\frac{1}{2\lambda}, \quad z = -\frac{1}{4\lambda} \quad \dots \textcircled{1}$$

また、は

$$\lambda = -\frac{1}{3}, \quad x = \frac{3}{2}, \quad z = \frac{3}{4} \quad \dots \textcircled{2}$$

$$\textcircled{1} \text{ のとき } g = 0 \quad \text{より}$$

$$\frac{1}{4\lambda^2} + \frac{1}{8\lambda^2} = 1 \quad \therefore \lambda = \pm \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\therefore z = \left(\mp \frac{\sqrt{2}}{\sqrt{3}}, 0, \mp \frac{1}{\sqrt{6}} \right) \text{ のとき } f = \mp \sqrt{\frac{3}{2}}$$

$$\textcircled{2} \text{ のとき } g = 0 \quad \text{より}$$

$$\frac{9}{4} + 3y^2 + \frac{9}{8} - 1 = 0$$

$$y^2 = -\frac{19}{24}$$

解なし

$$\text{よって } \left(-\frac{\sqrt{2}}{\sqrt{3}}, 0, -\frac{1}{\sqrt{6}} \right) \text{ のとき 最小値 } -\sqrt{\frac{3}{2}} \text{ をとる.}$$